

HIGH RESOLUTION MODELLING OF TIDAL RESOURCES, EXTRACTION AND INTERACTIONS AROUND THE UK

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ABSTRACT

The Energy Technologies Institute (ETI) commissioned SMARTtide, a Continental Shelf Model of Northern European waters. Its principal aims were to assess the tidal energy potential around the UK, to inform the design of energy harnessing schemes, to understand the interaction between different tidal range and tidal current energy schemes, and to evaluate their impact on Northern European coasts. To that effect, coarse and detailed resolution versions of the model were developed.

The project considered a range of possible tidal schemes developed from now to 2050 in a range of combinations to assess interactions. Key findings of the project highlighted that (1) prime current tidal sites can be identified, (2) tidal energy extraction can create far-field effects potentially impacting other arrays, and (3) there is a potential to optimise UK tidal resource development for any variable.

This makes SMARTtide a suitable tool for the tidal power industry to predict future tidal energy scheme scenarios, and the interaction between different energy schemes. SMARTtide will soon be publically available through a Fee for Service, hosted by HR Wallingford on behalf of the ETI.

BACKGROUND

The UK has established a global lead in the testing and deployment of tidal turbine devices, and multiple sites for single machines and proposed commercial arrays are established or under active development. Facilities such as the European Marine Energy Centre (EMEC) in Orkney have established testing and deployment best practice for tidal turbines and The Crown Estate have now leased multiple areas for early commercial array deployment (for example in the North Channel, Northern Ireland, Figure 1).



FIGURE 1. AREAS FOR TIDAL CURRENT DEVELOPMENT IN THE NORTH CHANNEL LEASED BY THE CROWN ESTATE.

As part of this rapidly developing industry, the ETI commissioned a project team led by Black & Veatch and supported by HR Wallingford and the University of Edinburgh to undertake tidal

resource modelling, which led to the development of the SMARTtide models:

- *Coarse Continental Shelf Model (CCSM)* for a high level review and initial evaluation of tidal schemes,
- *Detailed Continental Shelf Model (DCSM)* for a feasibility assessment of project development, and
- Bristol Channel *Detailed Tidal Range Model (DTRM)* for assessment of tidal barrage and lagoon schemes in the Severn Estuary.

The project was commissioned to look at the interactions between tidal current/turbine and tidal range/barrage energy schemes and to investigate cumulative and in combination effects. Both technologies are under active consideration in the UK and the cumulative impacts of build out of different scenarios in decades up to 2050 were considered in detail for their far-field and near-field effects [18].

The ETI has subsequently contracted HR Wallingford to host the model for use by the tidal energy industry and the UK Government bodies involved in the industry. The model has been developed into the SMARTtide brand as part of this commercial application following the successful conclusion of the development phase of the project.

METHODOLOGY

The SMARTtide suite of models is based on the open source, industry driven, TELEMAC system (www.opentelemac.org). TELEMAC was originally developed by EDF (France), and HR Wallingford has over 18 years' experience in using this model for complex flow modelling solutions. The 2D model can be obtained at no cost from the opentelemac website.

The superiority of the TELEMAC system resides in part in the use of unstructured grids of triangles to represent the model area. This allows the grid size to vary smoothly (a growth rate in the grid size of 8% was used here) for a detailed representation of areas of interest and coarse representation of open/deep water areas.

Another sizeable advantage of the TELEMAC system resides in the parallelisation of its code. This allows CPU intensive simulations to be carried out in a timely manner, if split on several cores on a HPC system. For example, the CCSM (c. 160 000 nodes) ran 17 days in 35 minutes using 20cores, and the DCSM (c. 1 600 000 nodes) ran 17 days in 1h45 using 72 cores.

Resolution, extent, seabed map and tidal forcing of the model

SMARTtide comprises three models, which extent and resolution (or grid size) are defined by their intended use.

The CCSM not only covers the UK waters but extends offshore slightly beyond the Northern European continental shelf (defined principally by the 300 m depth contour) in order to include all (potential) sites and to cater for long-range impacts and interactions between energy schemes. Recent publications [4, 7] have indicated that relatively small tidal power projects can affect very distant locations. It includes the coastlines of the United Kingdom, Ireland, the Channel Islands, France, Belgium, the Netherlands, Germany, Denmark, Sweden and Norway. It includes, amongst others, the Malin Sea, Irish Sea, Celtic Sea, English Channel and the North Sea. The Baltic Sea is not included in the model because of its very limited tidal range and minimal mean spring tidal current velocities [6]. An annual mean discharge from the Baltic Sea is instead imposed as an inflow in the model. The extent of the CCSM is shown in Figure 2. Its grid size varies between 1 km at the coastline, on islands and locations of selected tidal energy schemes, and 35 km in open water.

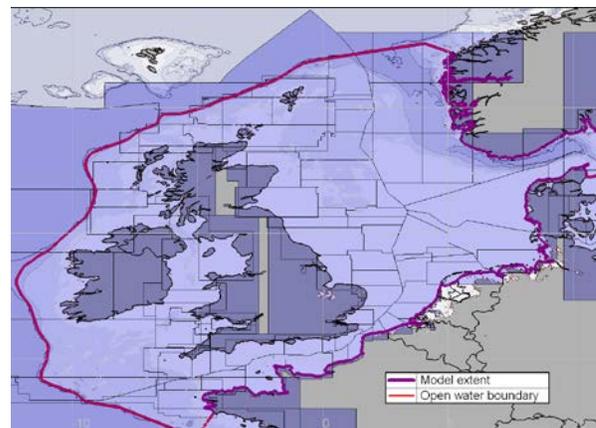


FIGURE 2. SMARTtide CONTINENTAL SHELF MODEL EXTENT AND OUTLINES OF NAVIGATION CHARTS INCLUDED IN THE MODEL.

The DCSM coverage is identical to that of the CCSM, but it is significantly more resolved. The grid size varies between 200 m at the coastline, on islands and energy scheme sites, and 10 km in open water, away from these sites.

The Bristol Channel DTRM is a local model, interfacing with the DCSM at the mouth of the Severn Estuary. Its grid size varies between 50 m

at pre-defined areas of interest and 2 km at the interface with the DCSM.

The resolution thresholds for each of the model grids were defined such that passages, inlets, islands and any other small-scale features would be adequately represented in the models. In addition, significant effort was invested to identify and group clusters of small islands into larger land masses, in order to represent the hydrodynamics as closely as possible. In the example illustrated in Figure 3, the Isles of Scilly, off the south-westernmost tip of England, were individually contoured in the DCSM (purple) but clustered, to some extent, in the CCSM (orange).

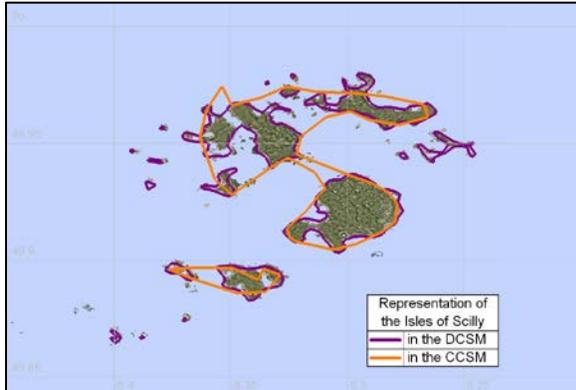


FIGURE 3. REPRESENTATION OF THE ISLES OF SCILLY IN THE MODELS.

It is noted that, while the resolution of the DCSM provides more detailed predictions than that of the CCSM, its purpose, like the CCSM, is primarily to provide preliminary impact assessment results for the entire Northern European continental shelf. It is not intended to be used in place of a refined local model for resource / impact assessment in specific areas.

The bathymetry in the models was developed from processed Admiralty Chart data. The outlines of the navigation charts included in the models are depicted as black geometrical shapes in Figure 2.

The SMARTtide models are driven by spatially varying time histories of water levels along the model offshore boundaries. A radiative boundary treatment [10, 16] allows waves to leave the model area with little or no reflection. This ensures that the tidal boundary conditions are still appropriate should the effects of the implementation of tidal energy schemes reach the model boundary.

The water level time histories in the CSMs (CCSM and DCSM) were derived from tidal synthesis based on the harmonic constituents available from the TPXO dataset. TPXO is one of

the most accurate global models of ocean tides [15]. It is based on a best-fit of tidal levels measured along remote sensing tracks from the TOPEX/POSEIDON satellite project in operation since 2002. The constituents derived from complex harmonic analysis of observed levels are by definition clean of atmospheric and surge variations, as these variations would not be part of the astronomical periods against which the tidal harmonics are being predicted.

For the DTRM, the water level time histories were extracted from the corresponding DCSM run.

A 15-day period featuring above average spring conditions and below average neap conditions, was used to calibrate and validate the CSMs to ensure good performance of the models for the entire range of expected tidal conditions. A 20-year average spring-neap 15-day cycle was subsequently used to model different combinations of tidal current and tidal range energy schemes.

Calibration, validation and verification of the CSMs

The data used to calibrate and validate the CSMs comprised two independent sets of observed coastal and offshore tidal gauge and bottom pressure data. Calibration was achieved by tuning the bed friction parameter until good overall agreement was reached at 24 coastal tidal gauges throughout the model area [9, 12, 17].

These locations were selected such that (a) they cover the entire model area (this is particularly relevant since one of the principal aims of SMARTtide is to inform the impact of the implementation of tidal energy schemes upon other interests); (b) they represent the possible range of expected spatial variations in tidal amplitude throughout the model area; and (c) they are located close to key areas of interest (e.g. Hinkley Point – Avonmouth in the Bristol Channel, Portrush in the North Channel).

It is important to note that the bed friction parameter is only a representation of bed features and bed properties in a complex model like TELEMAC. It does not account for any other physical process and as such, there is no need to validate the model with power extraction.

Agreement of the CSMs predictions with observed data was illustrated by comparison of the predicted and observed water level traces over the full 15-day period. While the time histories give an immediate visual impression of the agreement, the quality of the calibration was

assessed by computing the Root Mean Square Error (RMSE) throughout the 15-day tidal cycle.

The RMSE values were subsequently normalised with respect to the maximum tidal amplitude at the calibration site. It is our experience that a normalised RMSE value of 10% reflects a good calibration. The values achieved in the CCSM and in the DCSM are depicted in Figures 4 and 5 respectively as coloured spots. For example, dark and light green spots indicate locations where the normalised RMSE was below 5% and 10% respectively.

A similar analysis was repeated for the CSMs validation against independent sets of observed offshore tidal gauge and bottom pressure data (at 11 stations). This analysis confirmed the suitability of the CSMs throughout the Northern European continental shelf and in key energy areas in particular.

The data used to verify the models comprised velocity data and atlases of tidal range and peak current speed. Although the agreement of the CCSM with velocity data was mixed (principally because of its coarse resolution), the DCSM velocity predictions compared very favourably with measurements available at the time of the study.

Examples in the North Channel and off the coast of Anglesey are shown in Figures 6 and 7 respectively. To protect the copyright attached to the measured velocity data, the analysis is presented in terms of normalised values (with respect to the spring peak value at that location during the verification period).

Verification against the MAFF Atlas [14] was successful with the amphidromic points and the areas of high tidal range qualitatively very well reproduced in the models.

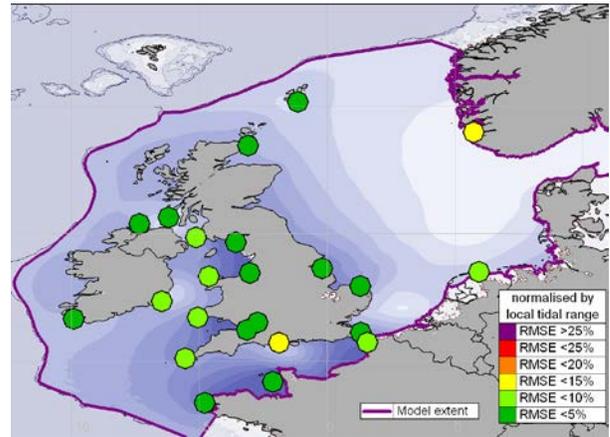


FIGURE 4. QUALITY OF THE CCSM CALIBRATION EXERCISE MEASURED IN TERMS OF NORMALISED RMSE.

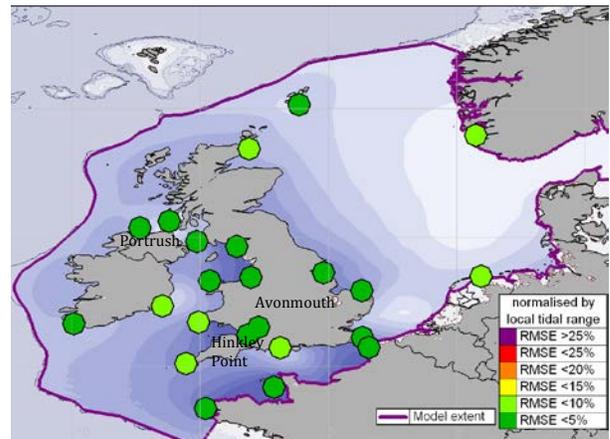


FIGURE 5. QUALITY OF THE DCSM CALIBRATION.

Verification against the UK Marine Renewable Energy Resources Atlas [8] (Figure 8 for example) was also positive for known energetic areas. It is noted that the finer resolution of the CSMs (compared to that used in [8]) allows a far better resolution of the velocity field in key areas. This is illustrated in Figure 8. As such, the DCSM (and to a lesser extent the CCSM) identified strong current areas that had been previously misrepresented in the UK Atlas.

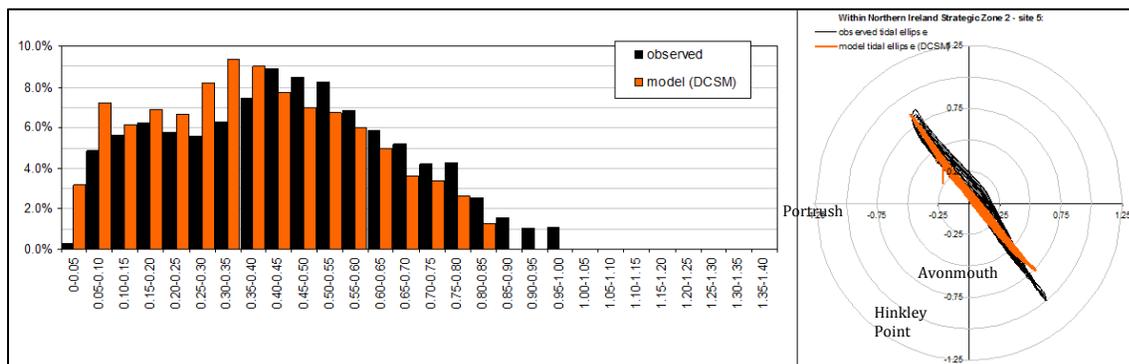


FIGURE 6 QUALITY OF THE DCSM VERIFICATION IN THE NORTH CHANNEL.

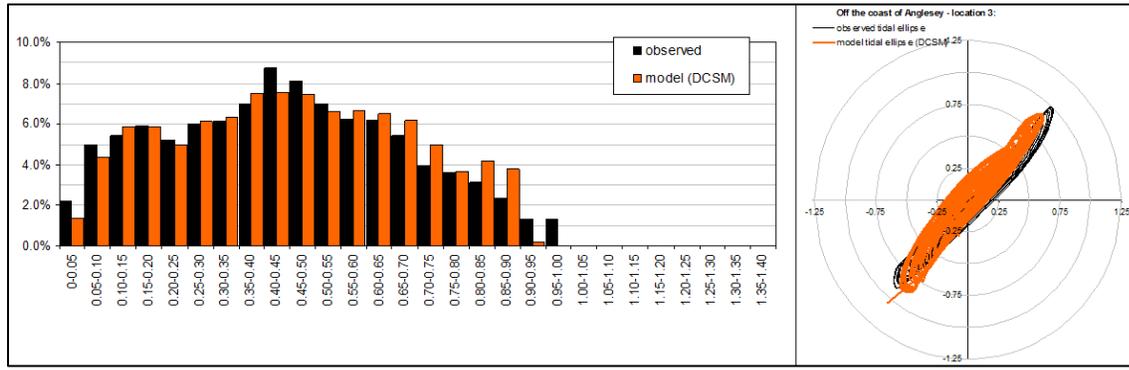


FIGURE 7 QUALITY OF THE DCSM VERIFICATION OFF THE COAST OF ANGLESEY.

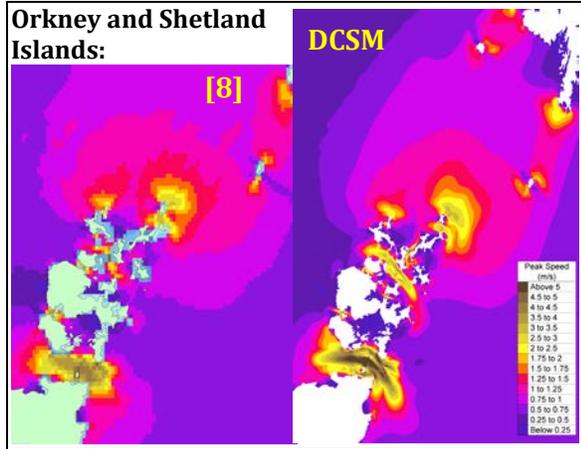


FIGURE 8. VERIFICATION OF THE DCSM AGAINST THE UK ATLAS [8].

These comparisons against atlases, as opposed to the spot checks performed in the calibration/validation exercise, enhance the overall credibility of the SMARTtide models.

It is reminded that the CCSM is intended for high level tidal range and/or tidal current investigations; the DCSM for investigation of tidal current schemes, as the greater resolution predicts tidal currents (and spatial variability thereof) more accurately. The DCSM is also intended for detailed site investigations into tidal range schemes, as more detailed bathymetry can be incorporated in the model. Neither model, however, is intended to replace a local detailed model for the final stages of optimisation and design.

Parameterisation of energy extraction

The primary purpose of SMARTtide is to provide a tool to assess the tidal energy potential around the UK, to inform the design of energy harnessing schemes and to evaluate their impact on European coasts.

TELEMAC-2D, the 2D module of the TELEMAC system on which SMARTtide is based, solves the 2D depth-averaged shallow water equations, also called the St Venant equations [11]. These comprise three equations (Equation 1 for the conservation of the volume of water and Equations 2 and 3 representing the conservation of the water momentum) dependent on three environmental hydrodynamic variables (the water depth h (m) and the depth-averaged velocity component u and v (m/s)). These equations are as follows:

$$\frac{\partial h}{\partial t} + \frac{\partial hu}{\partial x} + \frac{\partial hv}{\partial y} = Srce \quad (1)$$

where $Srce$ is a variation of the volume of water within the water column (including rain, evaporation and other intakes and outlets such as found around hydraulic structures) and where $\partial/\partial t$, $\partial/\partial x$ and $\partial/\partial y$ are respectively the time and space component gradients; and

$$\rho h \frac{\partial u}{\partial t} + \rho hu \frac{\partial u}{\partial x} + \rho hv \frac{\partial u}{\partial y} = -\rho gh \frac{\partial(h+b)}{\partial x} + \frac{\partial}{\partial x} \left[\rho hv_e \frac{\partial u}{\partial x} \right] + \frac{\partial}{\partial y} \left[\rho hv_e \frac{\partial u}{\partial y} \right] + hF_x \quad (2)$$

$$\rho h \frac{\partial v}{\partial t} + \rho hu \frac{\partial v}{\partial x} + \rho hv \frac{\partial v}{\partial y} = -\rho gh \frac{\partial(h+b)}{\partial y} + \frac{\partial}{\partial x} \left[\rho hv_e \frac{\partial v}{\partial x} \right] + \frac{\partial}{\partial y} \left[\rho hv_e \frac{\partial v}{\partial y} \right] + hF_y \quad (3)$$

where F_x and F_y are source terms and body forces acting on the water momentum (including seabed friction, Coriolis, drag, and possible energy extraction devices), and where g is the earth gravitational acceleration (m/s^2), v_e a diffusion coefficient (m^2/s) including dispersion and turbulence, ρ the water density (kg/m^3) and b the seabed elevation above datum (m).

Generic terms were implemented in TELEMAC-2D to model the effect of both tidal range and tidal current energy devices:

- The discharge through a tidal range scheme, Q , is defined as a water sink upstream of the turbines, and a source downstream of the turbines (*Src* in Equation 1, subroutine PROSOU). It is a function of the head or water level difference across the control structure and can be defined as the sum of several contributions, as follows:

$$Q = Q_{orifice\ sluices} + Q_{pumping} + Q_{free-running\ turbines} + Q_{generating\ turbines} \quad (4)$$

Not all terms apply depending on the particulars of the scheme being tested. As an example, the discharge through generating turbines, $Q_{generating\ turbines}$, can be parameterised using cubic polynomial equations based on the turbine hill chart:

$$Q_{generating\ turbines} = B_1 + B_2H + B_3H^2 + B_4H^3 \quad (5)$$

where H is the head difference across the barrage (m), and B_1 to B_4 are the discharge-head characteristics, functions of the technology type, the operational procedures, and other known key turbine parameters.

The head or water level difference across the barrage, on which Equation 4 is based, is computed dynamically and is constantly updated by the model. Thus the discharge (and by extension the power available) varies with the head or water level difference.

- When tidal current devices are introduced in the hydrodynamic system, the system loses energy, whether the energy is extracted or whether drag forces are introduced. The various contributions for the loss of energy are represented as additional drag terms (subroutine DRAGFO), the sum of which will be F_x and F_y in Equations 2 and 3:

$$F_e = -[power\ curve]/\|U\| \quad (6)$$

$$F_d = -1/2 \rho \|U\|^2 C_D A \quad (7)$$

where F_e corresponds to the energy extracted from the system and F_d to the drag exerted by the devices. *power curve* is the power curve specific to the device being tested ($\text{kg m}^2\text{s}^{-3}$) (an input to the models), $\|U\|$ is the norm of the depth-averaged velocity (m/s), ρ is the water density (kg/m^3), C_D is a structural drag coefficient and A the associated area (m^2). These additional momentum terms are a means of parameterising physical processes that occur at higher resolution than is used within the model. The parameterised terms replace small-scale physical processes (from the point of view of model resolution) with a continuous property applied across computational cell.

SMARTtide FOR THE TIDAL POWER INDUSTRY The Web User Interface

From the outset of this project it was the intention that the models would become available to the tidal power industry, through a Fee-For-Service arrangement, to understand the possible interactions between proposed tidal energy schemes across Northern European waters. Under this arrangement, registered users of SMARTtide are able to remotely operate the suite of tools: CCSM, DCSM, and/or DTRM.

A Web User Interface was, therefore, developed that allows Users to upload a tidal energy scenario of their design. A scenario may comprise one or many individual tidal energy schemes. The schemes may include both tidal current and tidal range schemes, although they are entered in to the models using different input methods. The schemes can be located close to each other or anywhere within the boundaries of the SMARTtide model.

A tidal scheme is defined by:

- a geographical extent and location, described as an open polyline representing the barrage or lagoon alignment, or as a closed polygon representing the area within which the tidal current turbines are to be sited; and
- parameters defining how SMARTtide should respond to the presence of the scheme, summarised in a formatted text file.

A tidal range scheme is characterised by: (a) a mode of operation (ebb-only generation, flood-only generation, and dual (ebb-flood) generation); and (b) a set of numerical parameters. These

parameters include turbine and sluice characteristics, pump characteristics, packing, as well as cubic polynomial functions which represent head-discharge-power characteristics, and turbine operating rules. These parameters inform Equation 4.

The technology options reviewed in the original study included conventional turbines (Bulb and Straflo) and a new low head turbine. By design, any type of turbine can be catered for with the generic parameterisation used in the model and introduced in Equation 4.

A tidal current scheme is characterised by: (a) a number of devices per km² within the scheme area (or a device footprint in m², whichever is readily known); (b) a structural drag coefficient and associated support structure area, which both depend on the technology; and (c) a power curve or thrust curve for extracted energy, which also depends on the technology. These parameters inform Equations 6 and 7.

It is important to consider that SMARTtide was developed as a set of 2D tools as a starting point, and therefore, uses a depth-averaged velocity as its basis. Dependent on the desired location of the support structure and turbines within the 3D domain, and the expected velocity profile of the site with depth, it may be necessary to adjust the parameters used to characterise the tidal current scheme in the models.

Behind the scenes, a project developed using Jenkins, an open source continuous integration tool, and Proteus, a data transformation tool, ensures that (1) the input received from the User are valid, (2) submission of the scenario automatically triggers a TELEMAC-2D simulation, (3) the simulation output are post-processed for end-user consumption and (4) the results are sent back to the User for analysis in a GIS package. It is noted that water depth and current velocity components are the primary variables of TELEMAC-2D. Post-processing steps are performed on these to provide changes (relative to baseline scenario including only La Rance tidal power plant) in maximum tidal range (m), mid-range speed (m/s) and average kinetic power density (kW/m²).

Example applications

Not detailed in this paper, but the subject of [18], the CSM was employed to investigate a number of viable options for tidal energy projects around the UK, introduced in [5] and

corresponding to a real interest in terms of tidal power resource.

Figures 9 and 10 present examples of such implementations in the DCSM. In these figures, the tidal current/range schemes are identified as black dashed lines; the impact is expressed in terms of tidal range difference (%) in Figure 9, and in terms of mid-range speed changes (%) in Figure 10, relative to the baseline scenario.

In the case depicted in Figure 9, 120 conventional turbines were sited along the lagoon embankment (southernmost scheme) for a total installed capacity of 3600 MW; and 1065 low-head turbines were sited in the barrage, across the estuary, for a total installed capacity of 5130MW. A significant reduction in maximum tidal range (pink areas) was observed upstream of the longest barrage following its construction. The smaller barrage yielded a lesser, yet noticeable, reduction (deep blue areas). This configuration did not conform to the expressed requirement to maintain at least 80% of the natural tidal range in the basin, and alternative options were considered.

In the case depicted in Figure 10, two tidal farms were developed in relatively close proximity with total installed capacities of 72 MW and 280 MW respectively. This case clearly highlighted problematic intra-scheme interactions with some of the devices in the wake of others.

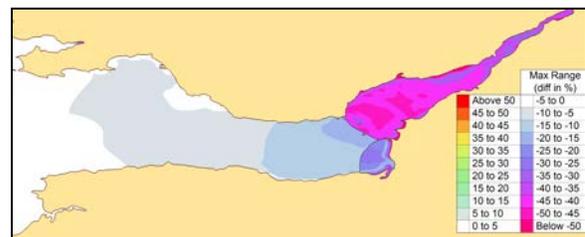


FIGURE 9. EXAMPLE IMPACT OF TWO TIDAL RANGE SCHEMES IMPLEMENTED IN THE DCSM. EXPRESSED IN TERMS OF MAXIMUM RANGE DIFFERENCE (%).

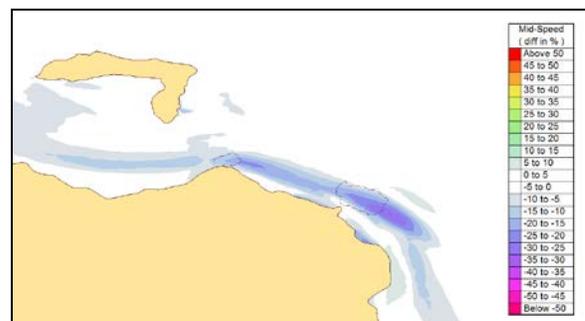


FIGURE 10. EXAMPLE IMPACT OF A TIDAL CURRENT SCHEME IMPLEMENTED IN THE DCSM. EXPRESSED IN TERMS OF MID-RANGE SPEED CHANGE (%).

CONCLUSIONS

The results of the SMARTtide model development have shown that it is possible to build a wide area highly resolved model of tidal energy extraction, accurately represent interactions between tidal energy arrays / barrages and to understand the impacts their installation has on tidal range and velocity across a very wide area. The resolution of the UK continental shelf model is finer than any previous model developed at this scale and this is a unique and powerful tool for planning the deployment and consenting of tidal turbine arrays in the UK (and equally applicable to the northern coast of France where there is more resource than previously estimated).

The same approach would be possible to adopt for other geographic areas where tidal energy resources could be exploited by more than one developer in the area of interest. Significant areas of tidal resource are known to exist in areas such as the Bay of Fundy / Maine and around the Korean Peninsula. Active consideration of use of these is under way with test sites analogous to EMEC in Scotland being constructed or planned. Accurate modelling of the tidal energy extraction and potential interactions between developments will be necessary if the optimum use of a constant and predictable energy resource for low carbon electricity are to be achieved. The inputs for such a model in terms of bathymetry, tidal harmonics and calibration data are relatively easy to obtain and proven methods have been used to construct the SMARTtide model.

ACKNOWLEDGEMENTS

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