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USING EXTENDED KALMAN FILTERS FOR REAL-TIME ESTIMATION OF EXCITATION FORCES ON A WAVE ENERGY CONVERTER

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INTRODUCTION

As the demand for renewable energy sources increases, the possibility of adapting wave energy as one of those sources becomes more enticing. It has many potential benefits over other renewable energy sources, including reduced variability and better predictability [1]. However, until its cost of energy is reduced enough to be competitive, it will not likely be widely adopted by utilities and power producers.

One option to decrease the cost of wave energy is to increase how much power wave energy converters (WECs) produce by applying active control. There has been a fair amount research on the optimal control of WECs (e.g. [2]–[5]), but much of this work depends on having accurate estimates of the current and future excitation forces on the WEC. Without knowledge of the excitation force, many of the proposed control schemes cannot be implemented.

The work described in this extended abstract specifically addresses the problem of estimating the current excitation force on a generic one-body point absorber WEC. The problem is treated as a disturbance estimation problem encountered in control-system design. A linear Kalman filter and multiple extended Kalman filters (EKFs) are simulated. The wave data that was used as an input for the simulation is measured time series data from near the Oregon Coast.

The next section presents the model used to simulate the motion of the WEC. Then the disturbance estimation schemes are presented. The abstract concludes with a presentation of the simulation results.

DYNAMIC MODEL

The WEC modeled is a generic one-body heaving point absorber. For simplicity, the body is assumed to be a cylinder. Additionally, we consider that the WEC configuration implies the power take-off force also acts as the mooring force. This yields the following time domain equation of motion:

$$m\ddot{z} = -F_b - F_r + F_e + F_{\text{PTO}} \tag{1}$$

where z is the heave position of the device, F_b is the buoyancy force, F_r, is the radiation damping force, F_e is the excitation force, and F_{PTO} is the power takeoff force.

Assuming linear wave theory, neglecting viscous forces, and assuming small wave heights, the radiation force and excitation force can be determined by convolving their impulse response functions with velocity and water surface elevation respectively.

The excitation force can then be expressed as

$$F_e(t) = \int_{-\infty}^{\infty} \mathbf{h}(\tau)\,\eta(t-\tau)\,d\tau \tag{2}$$

where $\mathbf{h}$(t) is the impulse response function of the excitation force, and η is the water surface elevation at a reference point. For this work, the frequency-domain results were calculated in ANSYS Aqwa [6]. These frequency-domain results are then used to calculate the impulse response functions using the inverse discrete Fourier transform. For more details on determining the impulse response functions used to model WECs see [7].

The buoyancy force is given by

$$F_b = \rho g S z, \tag{3}$$

where ρ is the water density, g is the gravitational constant, and S is the cross-sectional surface area of the mean water line.

The power takeoff is assumed to be a simple generator with a constant damping ratio. This yields

$$F_{\text{PTO}} = -b_g \dot{z} , \tag{4}$$

Where b_g is the constant damping ratio.

The radiation force is typically calculated by convolving the radiation impulse response function $\mathbf{k}$(t) with the device's velocity.

$$F_r = \int_t^{\infty} \mathbf{k}(\tau)\, \dot{z}(t-\tau)\, d\tau - A_{\infty} \ddot{z} \tag{5}$$

However, in the work presented here a reduced order state space model of order three is used to calculate the convolution term in the radiation force, shown in Equation (6).

$$\begin{aligned} \dot{\zeta} &= A_r \zeta + B_r \dot{z} \\ F_r^{'} &= C_r \zeta \end{aligned} \tag{6}$$

In the reduced order state space model, ζ is the radiation state vector, and $A_r, B_r,$ and C_r are determined using the process outlined in [8].

Collecting all of these forces, the full equations of motion can be written in state space form

$$\dot{x} = \begin{bmatrix} A_r & 0 & B_r \\ 0 & 0 & 1 \\ \dfrac{-C_r}{m+A_\infty} & \dfrac{-\rho g S}{m+A_\infty} & \dfrac{-b_g}{m+A_\infty} \end{bmatrix} x + \begin{bmatrix} 0 \\ 0 \\ \dfrac{1}{m+A_\infty} \end{bmatrix} F_e(t) \tag{7}$$

$$y = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} x \tag{8}$$

where $x = [\,\zeta \;\; z \;\; \dot{z}\,]^T$ and all zeros are appropriately sized vectors or matrices. The excitation force is calculated for the full time series before running the simulation, and is then treated as an exogenous input to the system.

DISTURBANCE ESTIMATION

The Kalman filter is a widely used approach to estimate the state of a dynamic system with noisy measurements and model uncertainties. It is often used in real time state estimation applications due to its computational efficiency [9]. For a linear system with Gaussian white noise, the Kalman filter is the optimal state estimator [9]. A system model is used to predict what the next state will be, and then the predicted state is corrected with the measurement data. The discrete Kalman filter is used in the work presented here. To use this approach, the form of the dynamic system must be

$$\begin{aligned} x(k+1) &= A_k x(k) + B_k u(k) + v_k , \\ y(k) &= C_k x(k) + w_k , \end{aligned} \tag{9}$$

where x is the system state vector, y is the available measurements, and v and w represent the noise in the process and measurements respectively. The noise vectors v and w have covariance matrices Q and R respectively. If A and B depend on x, the Extended Kalman filter can be used for the nonlinear system. Note that in the Kalman filter formulation, it is assumed that u is known exactly. Since we are not implementing any control in this work, $u(k)$ does not exist for any of the disturbance estimators presented.

A Kalman filter estimates the state of a dynamic system, so any value that needs to be estimated must be an element in the state vector x. To use a Kalman filter to estimate disturbances, we need to augment the system to include the estimated disturbance as additional state variables.

We present four different models to be used to estimate the current heave excitation force from measurements of heave position and velocity. First a linear model is presented, followed by three nonlinear models. Details on implementing both linear and Extended Kalman filters can be found in [9].

Linear Kalman Filter

The first proposed prediction model to be used assumes a simple harmonic oscillator model for the excitation force. We also simplify the radiation force model, assuming it is linearly proportional to heave velocity. This simplification is done to minimize the number of states in the prediction model. This yields the following A matrix:

$$A^{(1)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \dfrac{-k}{m_{\text{tot}}} & \dfrac{-\tilde{b}}{m_{\text{tot}}} & 0 & \dfrac{1}{m_{\text{tot}}} \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\tilde{\omega}^2 & 0 \end{bmatrix} \tag{10}$$

where the state vector is $x = [\, z \;\; \dot{z} \;\; F_e \;\; \dot{F}_e \,]^T$, $\tilde{b}$ is the assumed radiation coefficient, $k = \rho g S$, the hydrostatic constant, $M_{tot} = m + A_\infty$, and $\tilde{\omega}$ is the assumed frequency of the excitation force. This system model is given in continuous time, so it is converted to a discrete model and then used to implement a linear Kalman filter. Values of $\tilde{\omega}$ and $\tilde{b}$ are determined by manually tuning to minimize prediction error for a variety of test cases. The final selected values are shown in Table 1.

TABLE 1. ASSUMED PARAMETERS FOR LINEAR ESTIMATORS.

Parameter	Value
$\tilde{b}$	$40\ ^{\mathrm{kNs}}/_{\mathrm{m}}$
$\tilde{\omega}$	$2\pi \frac{1}{9}\ ^{\mathrm{rad}}/_{\mathrm{s}}$

The standard Kalman filter implementation assumes the process and noise covariance matrices (Q and R) are known. In reality these covariance matrices need to be estimated. For this work the measurement noise covariance, R, was assumed to be known from the noise statistics, and the process noise covariance, Q, was tuned to maximize estimation accuracy over a wide range of sea states. Since the true excitation force values are known from the dynamic simulation of the WEC, Q was chosen to minimize estimation error. This tuning was done manually to avoid over fitting the model to the subset of data that was used to tune the estimators.

Extended Kalman Filter

To eliminate the need to choose values of $\tilde{\omega}$ and $\tilde{b}$, we can instead treat one or both of these constants as additional state variables in our system model. This allows the Kalman filter to adaptively change the assumed values to hopefully improve estimation accuracy. However, this also makes the system nonlinear.

Three nonlinear estimators are investigated in this work. The first keeps $\tilde{b}$ constant while estimating $\tilde{\omega}$, and is referred to as EKF$^{(2)}$. The second estimates $\tilde{b}$ while assuming a constant $\tilde{\omega}$ (EKF$^{(3)}$), and the third estimates both $\tilde{b}$ and $\tilde{\omega}$ (EKF$^{(4)}$). The "A" matrix for the first EKF case is

$$A^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ \frac{-k}{m_{tot}} & \frac{-\tilde{b}}{m_{tot}} & 0 & \frac{1}{m_{tot}} & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -x_5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

where the state vector is now $x = [\, z \;\; \dot{z} \;\; F_e \;\; \dot{F}_e \;\; \tilde{\omega}^2 \,]^T$. The last row is a zero vector because the rate of change of the frequency is assumed to be zero. By including a small amount of process noise the value can change gradually over time. The same process is followed to create $A^{(3)}$ and $A^{(4)}$. A$^{(3)}$, which estimates $\tilde{b}$, is the same as $A^{(2)}$ except the (2, 2) entry is $-x_5$, and the (4, 3) entry is $-\tilde{\omega}^2$. $A^{(4)}$, which estimates both $\tilde{b}$ and $\tilde{\omega}$, is a 6 by 6 matrix. $A^{(4)}$ is constructed by appending a zero row and zero column to the bottom and right of $A^{(2)}$. Then the (2, 2) entry is changed to $-x_6$. The full $A^{(4)}$ and $A^{(4)}$ are not shown for brevity. For extended Kalman filters that estimate only one parameter, the value for the constant parameter used is the same as was used for the linear filter, shown in Table 1 Note the model as presented is a continuous time model.

These nonlinear models are then used with an extended Kalman filter to estimate the current wave excitation force. The extended Kalman filter uses the nonlinear system model to predict the state at the next iteration. Then the nonlinear model is linearized about the previous estimated state, and this linearized model is used in the standard Kalman filter equations. During implementation the continuous models shown here are discretized at each iteration for the extended Kalman filter. The linearization process at each step increases the computational complexity of the extended Kalman filters by a factor of approximately five over the linear Kalman filter. The same covariance tuning method was used for the extended Kalman filters as was used with the linear filter.

RESULTS

The motion of the WEC was simulated using time-series water surface elevation measurements from a Nortek AWACs underwater acoustic wave measurement device. The AWAC was located off the coast of Oregon in 40 meter water depth at the Pacific Marine Energy Center's North Energy Test Site. The dataset includes 601 40-minute time series' sampled at 2 Hz from Aug. 14 to Oct 3, 2014. Sea states observed range from significant wave heights between 0.47 to 6.73 m., and mean wave periods between 3.25 s. to 10.24 s.

The WEC's motion was simulated in the time domain using the ODE45 MATLAB function, a continuous time differential equation solver [10]. The input to the simulation was the recorded water surface elevation measurements. The simulation outputs are position and velocity of the simulated WEC motion with artificial Gaussian white noise with a standard deviation of 0.01 m in position and 0.005 m/s in velocity. The output of the system is at 10 Hz.

The discrete Kalman filters were then implemented using these measurements of position and velocity at 10 Hz. Performance of each estimator and each simulated time series was evaluated with the correlation coefficient between actual and estimated excitation force values. Note that the true values of excitation force are known from the dynamic simulation of the WEC, although these values are not used for estimation purposes. This comparison was made only with the values appearing at 2 Hz since that was the sampling time of the recorded water surface elevation. A correlation coefficient of 1.0 would indicate a perfect estimator, and anything less is indicative of less accurate estimations. Regression coefficients were used for comparisons instead of error measurements to more accurately compare estimations with different significant wave heights.

Figure 1 is a histogram showing the distributions of correlation coefficient values for each tested dataset and estimation filter. Additionally the median correlation coefficient for each estimator is shown in Table 2.

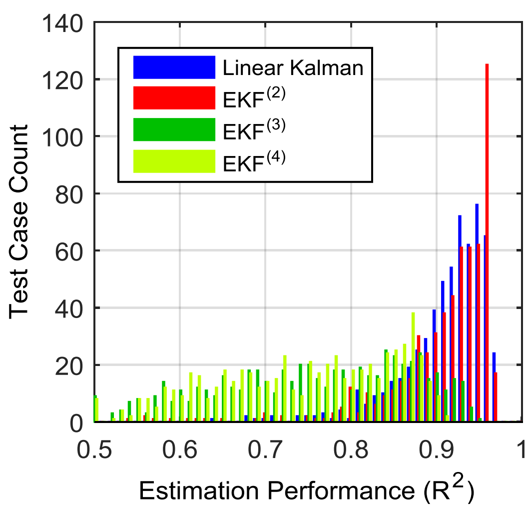

FIGURE 1. HISTOGRAM OF CORRELATION COEFFICIENTS FOR EACH ESTIMATION FILTER, WITH BIN WIDTHS OF 0.01.

TABLE 2. MEDIAN CORRELATION COEFFICIENTS FOR EACH ESTIMATION FILTER.

Kalman Filter	Median Correlation Coefficient (R^2)
Linear Kalman	0.9243
$EKF^{(1)}$	0.9293
$EKF^{(2)}$	0.7687
$EKF^{(3)}$	0.7606

The histogram plot shows the linear Kalman filter and the extended Kalman filter that dynamically estimated the excitation force frequency ($EKF^{(2)}$) performed much better on average than the other two extended Kalman filters. The regression coefficients for the first to estimators are closely concentrated between 0.9 and 0.97, while the coefficients for the other two extended Kalman filters are widely dispersed between 0.5 and 0.95.

The linear Kalman filter appears to perform slightly worse than $EKF^{(2)}$, and that is confirmed by looking at the median correlation coefficients. The difference between the two, however, is less than 1%.

Further investigation of the two best performing filters shows each has benefits in different sea states. Figure 2 shows the estimation correlation coefficients at each sea state of the linear estimator and $EKF^{(2)}$ vs. mean wave period. While the extended Kalman filter performs better on average for sea states with mid-range mean wave periods, the linear Kalman filter performs better on sea states with mean periods near 9 seconds and below 5 seconds. This is expected near wave periods near 9 seconds, as that is the assumed constant period used for the linear Kalman filter. It is somewhat unexpected that the linear filter outperformed the EKF at low mean periods since the EKF can theoretically adapt to the changing conditions while the linear filter has no way to adapt.

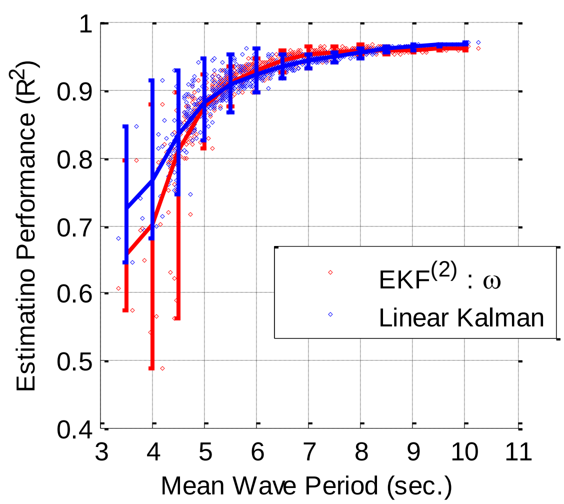

FIGURE 2. REGRESSION COEFFICIENTS FOR THE LINEAR ESTIMATOR AND $EKF^{(2)}$ VS. MEAN WAVE PERIOD. THE SCATTER POINT SHOW EACH SEA STATE, WHILE THE ERROR BARS REPRESENT THE MAX AND MIN VALUES WHEN GROUPED IN BINS 0.5 SECONDS WIDE.

The linear Kalman filter is likely outperforming $EKF^{(2)}$ on sea states with low mean periods due to the increased model inaccuracies of the WECs motion. The assumed damping ratio is a better approximation to the frequency-dependent radiation force at longer mean periods then sea states with shorter mean periods. As the assumed WEC model becomes more inaccurate, the Kalman filter adjusts the disturbance estimate to include force components from the inaccurate radiation force estimate, yielding a less accurate estimate of the excitation force.

The poor performance of $EKF^{(3)}$ and $EKF^{(4)}$ can be explained by the modeling inaccuracies of the simplified models. For $EKF^{(3)}$ the damping parameter is allowed to change. To improve estimation performance the constant damping ratio is adapted to improve estimation accuracy of position and velocity at the expense of the accuracy of the excitation force estimates. For $EKF^{(4)}$, it is likely the model has too many degrees of freedom for adaptation. The estimator will adapt to improve estimates of position and velocity, but in doing so the original structure of the model is lost. Too much adaptation leads to worse disturbance estimation accuracy.

The EKFs that performed poorly are adapting the model to improve the estimation accuracy of the measurable states at the expense of staying true to the intent of the original prediction model. Since the excitation force is not directly measurable the adaptations have no way to take this estimation accuracy into account when updating the parameters. The best wave excitation force estimator has to balance adaptability to different sea states with a rigid model structure developed with prior knowledge.

CONCLUSIONS

A dynamic model for the heave motion of a generic WEC for any water surface elevation input was developed using linear wave theory. This model was used to simulate the motion of the WEC under a wide variety of sea states encountered off the Oregon Coast. Assuming noisy measurements of position and velocity of the device are available, four estimators are presented to estimate the current excitation force on the WEC.

Out of the for estimators investigated simulation results showed that the linear Kalman filter and $EKF^{(4)}$ performed the best over a wide range of sea states. Both of these estimators provided a high level of accuracy over a wide range of sea states, with regression coefficients typically greater than 0.9. The extended Kalman filter provided only marginally more accurate predictions.

One additional challenge that would exist if this process was implemented on a real system is the lack of knowledge of the true excitation force. The accuracy of the estimations would have to be evaluated in simulation, or accuracies would have to be inferred from estimation performance of measurable values such as position and velocity.

This work is part of an ongoing research project investigating estimation and prediction methods for wave excitation forces on WECs. One main avenue of future work includes expanding the method to models with multiple degrees of freedom. While the basic approach for multiple degrees of freedom would remain similar, accounting for interactions between modes of motion would complicate tuning of the filters. Additional future work includes improving the process used to tune the covariance matrices of the Kalman filters, and investigating how the time interval of the Kalman filters effects estimation accuracy. We also plan on implementing a linear Kalman filter where the assumed constant values are dependent upon the current sea state.

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