CONSTRAINED OPTIMAL CONTROL OF SINLGLE AND ARRAYS OF POINT-ABSORBING WAVE ENERGY CONVERTERS

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ABSTRACT

A control method applied to a point-absorber wave energy converter is investigated by considering constraints on motions and forces in the time domain. The resulting performance of the wave energy converter under different sea states shows that this method leads to a latching-like performance and can highly improve conversion efficiency. The ongoing work for the control of arrays of devices is presented, and results show that the coordinated control of arrays can smooth the power output.

1. INTRODUCTION

Since the beginning of wave energy research in the 1970's, there have been attempts to improve the power production of wave energy converters (WECs) by optimizing the wave-structure interaction by different control methods [1-3]. For point-absorber WECs in long waves, one condition for maximizing the energy production is to keep the velocity of the heaving device in phase with the excitation force [4]. Latching is an example of a discrete control method where this phase condition is achieved by locking the motion of the device when the velocity is zero and releasing it again at proper times. Continuous phase control, also denoted reactive control, can be obtained by controlling the power take-off (PTO) device with a combined generator-and-motor. Reviews of comparisons between different control methods for wave energy devices can be found in, eg., [4-6].

Often, control strategies are discussed without considering the relationship between the PTO force and the converter's motion. This leads to two problems: one is that the PTO force can take any optimal value at any time, which is ideal and may lead to problems in the practical implementation of the control method. Another problem is that it is hard to separate the instantaneous active power from the total PTO power. In this paper, the PTO force is modeled as a function of velocity and displacement and consists of a spring and time-varying resistive load. In [6–8], the motion and forces were expanded in terms of truncated Fourier series and the optimal control problem of a single and arrays of WECs was transformed into a constrained finite dimensional optimization problem with convex quadratic cost function. Here, we follow their approach and extend it to constrained optimal control of the WEC by considering the relationship of PTO and motion of converter in the time domain, where PTO damping coefficient and PTO spring coefficient are defined to be non-negative and possible physical constraints are included in the solution of the optimization problem.

In order to generate electricity on a commercial scale and to reduce costs from mooring, maintenance, sea cable, and deployment, most wave energy concepts will have to deploy multiple devices in a wave energy farm, where the oscillation of each converter will produce radiated waves that influence each other. The interaction between the devices will affect the energy absorption of the devices, and can be exploited by control methods to optimize the performance of the full park. Coordinated control is one of that control methods and can improve the energy capture properties [8]. We extend the optimal control method to more converters with the same constraints used for a single WEC.

In section 2, we present this method, and point out the approximations and constraints used in this paper. Section 3 is a review for constrained optimal control of single devices, as presented in the recent paper [9,10]. The ongoing work to extend the control method to arrays of devices is presented and discussed in section 4.

2. METHOD

A point-absorber based on the wave energy device developed at Uppsala University is studied in this work [10]. The device consists of a floating cylinder connected to a linear generator on the seabed. If fluid flow is assumed to be inviscid, incompressible, and irrotational, then the heave motion of the WEC can be described by linear water wave theory as:

$$m\ddot{z}(t) = F_e(t) + F_r(t) + F_h(t) + F_{PTO}(t), \quad (1)$$

where m is the inertia mass of buoy and translator, z(t) the vertical displacement of buoy from equilibrium, $F_e(t)$ the excitation force, and $\mathrm{F_h(t)} = -\rho g \pi r^2 z(t)$ is the hydrostatic restoring force. Radiation force $\mathrm{F_r(t)} = -\mathrm{M}_\infty \ddot{z}(t) - \int_0^t \mathrm{K}(t-\tau)\dot{z}(\tau)d\tau$, and the radiation impulse response can be solved from Ogilvie's relation $\mathrm{K(t)} = \frac{2}{\pi}\int_0^\infty \mathrm{C(w)}\cos(\mathrm{wt})\,\mathrm{dw}$, where M_∞ is the added mass at infinite angular frequency and $\mathrm{C(w)}$ the radiation damping. The PTO force are represented by two components: a damping force proportional to the velocity and a spring force proportional to the displacement, namely

$$\begin{aligned}\mathrm{F_{PTO}(t)} &= \mathrm{F_{PTO_S}(t)} + \mathrm{F_{PTO_C}(t)} \\ &= -K_{PTO}z(t) - C_{PTO}(t)\dot{z}(t),\end{aligned} \quad (2)$$

where K_{PTO} is the spring coefficient, and $C_{PTO}(t)$ is the time-dependent damping coefficient which needs to be solved for. To avoid non-physical values, they should be defined as non-negative. Now, Equation (1) can be rewritten as

$$\begin{aligned}&(m + M_\infty)\ddot{z}(t) + \int_0^t K(t-\tau)\dot{z}(\tau)d\tau \\ &+(S + K_{PTO})z(t) = F_e(t) + F_{PTO_C}(t)\end{aligned} \quad (3)$$

The control problem can be defined as finding the optimal velocity or PTO coefficients to absorb the maximum energy over a long fixed time interval T by considering the constraints of the WEC, and the energy can be expressed as

$$\begin{aligned}E &= -\int_0^T F_{PTO}(t)\,\dot{z}(t)dt \\ &= -\int_0^T F_{PTO_C}(t)\,\dot{z}(t)dt.\end{aligned} \quad (4)$$

The velocity and the PTO force are assumed to be square-integrable functions in the interval [*0, T*] and can be written in terms of a Fourier series. By truncating the series, the problem is discretized and the expressions for the velocity and force are approximated as,

$$v(t) \approx \sum_{n=1}^{N} a_n \cos(nw_0t) + b_n \sin(nw_0t), \quad (5)$$

$$\mathrm{F_{PTO_C}(t)} \approx \sum_{n=1}^{N} c_{an}\cos(nw_0t) + c_{bn}\sin(nw_0t), \quad (6)$$

where a_n, b_n, c_{an}, c_{bn} are Fourier coefficients, w_0 the fundamental frequency, and similarly for the excitation force. Then an approximation of the solution to the equation of motion is solved using the Galerkin method. The result (for more details, see [7]) is

$$\mathbb{H}\boldsymbol{V} = \Phi + \boldsymbol{F}, \quad (7)$$

where $\boldsymbol{V}$, Φ, $\boldsymbol{F}$ are vectors of Fourier coefficients of velocity, PTO damping force and excitation force, and

$$\boldsymbol{V} = [a_1, b_1, a_2, b_2, \dots, a_N, b_N]^{\boldsymbol{T}}, \quad (8)$$

$$\Phi = [c_{a1}, c_{\boldsymbol{b1}}, c_{a2}, c_{b2}, \dots, c_{aN}, c_{bN}]^{\boldsymbol{T}}, \quad (9)$$

$$\boldsymbol{F} = [f_{a1}, f_{b1}, f_{a2}, f_{b2}, \dots, f_{aN}, f_{bN}]^{\boldsymbol{T}}. \quad (10)$$

The matrix $\mathbb{H}$ is block diagonal and its l_{th} block elements can be expressed as $\mathbb{H}^l = [C(lw_0), \delta\,;\,-\delta, C(lw_0)]$ with $l = 1,2,\dots,N$, where $\delta = \big(m + M(lw_0)\big)lw_0 - (S + S_{PTO})/lw_0$. The radiation resistances $C(lw_0)$ and the added masses $M(lw_0)$ are related to the impulse response.

By substituting Equation (5) and Equation (6) into Equation (3), the cost function of the resulting optimization problem can be written as $W = -\Phi^T\mathbf{V}$ [9]. If the matrix $\mathbb{H}$ is non-singular, the optimal Φ that maximizes the total converted energy can be obtained by solving the optimization problem,

$$\Phi_{\boldsymbol{optimal}} = \boldsymbol{arg}\,\mathbf{min}_\Phi[\Phi^{\boldsymbol{T}}\mathbb{H}^{-1}(\boldsymbol{F} + \Phi)] \quad . \quad (11)$$

Without considering the constraints on the motions or forces, the maximum power absorbed by the WEC can easily be found. However, the implementation of the constraints is vital to the practical converter, from the benefit of survivability and to make physical sense. In this paper, the physical constraints on displacement, velocity, and PTO damping force are considered, which can be expressed as

$$\begin{cases} |v(t)| \le v_{max} \\ |z(t)| \le z_{max} \\ |F_{PTO_C}| \le F_{PTO_C,max.} \end{cases} \quad (12)$$

Another constraint included in the model is the signs of the spring coefficient and damping coefficient, which are defined non-negative as follows

$$\begin{cases} K_{PTO} \ge 0 \\ C_{PTO}(t) \ge 0. \end{cases} \quad (13)$$

The issue is now converted to a minimization problem with a quadratic cost function and linear as well as nonlinear constraints. In a real time implementation, the optimization problem is calculated over a time horizon T, with a correct estimation of the excitation force. A control code is implemented using the active-set method in MATLAB. The constraints are imposed only at specified time instants $t_i = N_t * dt$ in the range $[0, T]$, N_t is an integer starting from zero and the time step is $dt = 0.1$ s. Hydrodynamic parameters are calculated using the boundary integral potential flow solver WAMIT. A prediction horizon of $T = 2\pi/w_0 = 62.8$ s and a Fourier length of $N = 80$ were found suitable in the current paper.

3. OPTIMAL CONTROL OF SINGLE DEVICE

In order to absorb maximum energy from the waves it is necessary to have optimum oscillation of the WEC. For a sinusoidal incident wave there is an optimum phase and amplitude for the oscillation. As shown in Figure 1, the excitation force is in phase with the velocity, at least approximately, and the displacement seems to be latched when the velocity vanishes, which is more obvious for long-period wave. In general, the control method leads to a latching-like performance, and produces more power.

The performance of a single WEC in sea states with wave amplitude of 0.3m, 0.4m and 0.5m can be found in our recent paper [10], the produced power exceeds the Budal & Falnes upper bound (BFUB) at low frequency, especially for large wave amplitude. In their derivation of the upper bound, Budal and Falnes assumed that a restriction of the design amplitude restricts the heave velocity of the buoy to $|v| < wV/2S_w$, where S_w is the water plane area. Here, this assumption does not hold due to the non-harmonic velocity of the buoy. Hence, the control method used in this paper implies that the mean power can exceed the Budal & Falnes upper bound. Pizer [11] and Hals [12] also reported that the power absorption can exceed the limit bound under the additional restriction of sinusoidal motion.

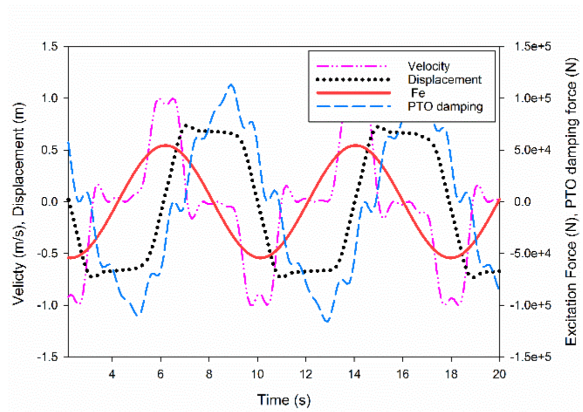

FIGURE 1. MOTIONS AND FORCES UNDER CONTROL WITH CONSTRAINTS. REGULAR WAVE WITH AMPLITUDE $A = 0.5\,m$ AND FREQUENCY $w = 0.8\,rad/s$. CONSTRAINTS ARE $z_{max} = 1\,m$, $v_{max} = 1\,m/s$, $K_{PTO} = 0\,N/m$.

On the other hand, even though the WEC has good performance at low frequencies, its capture width never exceeds the maximal capture width, which equals the wavelength divided by 2π (more details, see Figure 10 in [10]). At $w = 1.4$ rad/s, the capture width is close to the buoy's physical width, and the WEC has a high efficiency exceeding 90 percent. This phenomenon is more obvious in small amplitude waves. It also indicates that this control method can decrease the peak variation of power in short-period waves, which is represented by the ratio of peak power and mean power in different sea states.

4. OPTIMAL CONTROL OF ARRAYS OF DEVICES

Here, we present our on-going work for an array. The formulation for two WECs can be extended as

$$\begin{bmatrix} \mathbb{H}_{11} & \mathbb{H}_{12} \\ \mathbb{H}_{21} & \mathbb{H}_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}, \quad (12)$$

where $\mathbb{H}_{ij}$ is block diagonal and analogous to the H in Equation (7), V_i, Φ_i and F_i are the Fourier coefficients of velocity, PTO force and excitation force of i_{th}WEC. Now all the information of the farm is included in Equation (12). The optimal values can be obtained using the method introduced in section 3 if $\mathbb{H}$ is not singular.

The mean power produced by the arrays varies with wave climates and WEC characteristics [13,14]. Only the influence of considering or not considering the relationship of PTO force and WEC's motion is studied here, with $P_{m,c}$ and $P_{m,nc}$ representing the mean power in the two cases respectively. As shown in Figure 2, $P_{m,c}$ exceeds $P_{m,nc}$ by at least 20 percent, and this ratio can be as high as 60 percent when the wave length is 1.5 times the distance between the two WECs. Another phenomenon for the coordinated control is that it can smooth the output power of the whole array or farm. As shown in Figure 2, the ratio of peak instantaneous power and mean power varies in the range of [2.7, 9.3], which is also influenced by the distance between the WECs relative to the wavelength.

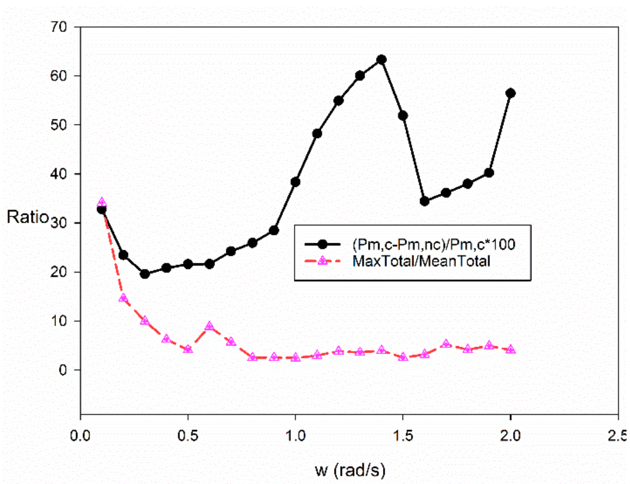

FIGURE 2. RESULTS FOR TWO WECs. The two WECs are located along the wave propagation direction, and the gap distance between them is 20 m. MaxTotal/MeanTotal is the ratio of the peak power and mean power produced by the arrays.

5. CONCLUSIONS

A time-domain analysis has been presented to evaluate the performance of a single and array of WECs oscillating in heave in regular waves. A control algorithm of the PTO has been studied by considering the constraints of motions and forces. It was found that, although no phase control has been used, this control method results in a latching-like behavior, and the excitation force are in phase with velocity. High conversion efficiency, more than 90% in some cases, can be achieved by this method. With this control method, the WEC has good performance in capture width, especially for long-period waves, where the mean power can exceed the upper bound stated in [4].

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